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TECHNICAL CHANGE AND THE RATE OF PROFIT: COMPARISON OF SRAFFA AND MARX

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The relationship between technical change and the rate of profit is one of the most debated issues in Marxian economics. This debate is concerned with whether there is a tendency for the rate of profit to fall (TRPF) as a result of capitalist accumulation, and therefore whether the very process of capitalist growth undermines the conditions of existence for continued accumulation.

In recent years the debate over the TRPF has focused on the Okishio Theorem. The Okishio theorem begins by claiming that rational, profit maximizing capitalists will only introduce viable technical changes. A viable technical change is one which earns 'super profits' for the innovating capitalist, i.e., is cost-reducing at the prevailing prices of production. The Okishio Theorem states that if a capitalist adopts a viable technical change, and once the effect of the technical change on relative prices has worked itself out, then the competitive, general rate of profit cannot fall. This result has had a profound effect on the debate over the TRPF. Indeed, for Philippe van Parijs the Okishio Theorem is so devastating for any claim that there is a TRPF " ... that it deprives all the arguments (pro and contra) of their relevance" (1980, p.9).

The Okishio Theorem is based on a number of explicit assumptions. These are: (1) there are n-single product industries, (2) there are no non-produced means of production, (3) there is no fixed capital, (4) real wages are fixed, and (5) all markets are competitive so that the profit rate, the wage rate, and the price of each commodity are uniform across the economy.

The Okishio Theorem also makes, usually implicitly, the strong assumption that the correct way to calculate the rate of profit is one where the rate of profit is defined in and through the Sraffian approach to value theory. Okishio made this point explicitly when he wrote "Marx calculated the general rate of profit as aggregate surplus value divided by aggregate capital in terms of value, that is m/c+v. But this procedure is not correct" (1961, p.90). Okishio then goes on to state that the correct rate of profit is the one employed in Sraffian linear price of production models.

The purpose of this paper is to demonstrate that the result of the Okishio Theorem depends critically on its use of a Sraffian value theory. I will show below that a viable technical change will lead to contradictory movements of the rate of profit when the rate of profit is calculated within the context of a Marxian value theory. This result will be demonstrated while maintaining all of the explicit assumptions, (1) - (5), of the Okishio Theorem as stated above. Thus, it is possible for the rate of profit to rise, calculated on the basis of Sraffian value theory, even though the Marxian rate of profit falls.

A word of epistemological caution is in order. The purpose of this paper is not to <u>disprove</u> or <u>extend</u> the Okishio Theorem. Instead, the argument developed in this paper is that the Okishio Theorem is proved on the basis of a Sraffian paradigm rather than a Marxian one. Therefore, the Okishio Theorem has <u>displaced</u> rather than disproved the traditional argument for the TRPF. Thus, the merits or demerits of each approach should be debated within the context of a 'paradigm strife', and not simply on the basis of different explicit assumptions.

This paper is organized as follows. The next section discusses the role which value theories play in economics in general, and then brings out some of the differences between Sraffian and Marxian value theories. The following section analyses the effect of a viable technical change on the Sraffian and Marxian rates of profit. Two models are developed there; one which assumes constant prices of production and then one which allows prices to adjust. It is demonstrated there that while the Sraffian rate of profit rises unambiguously (the Okishio result), the Marxian rate of profit can rise or fall depending on the relationship between the size of the elasticity of exploitation with respect to changes in productive labor and the size of the organic composition of capital. This paper will conclude with some general comments on the importance of value theory for theoretical and political analysis.

<u>Value Theory as Entry Point</u>

The social totality is complex and multifaceted. As in the parable of the blind man and the elephant, different social theorists will produce very different understandings of the social totality depending on where the social theorist begins to 'touch' or focus in on the elephantine social totality. In economics, this focus is defined, whether implicitly or explicitly, by the theorist's choice of a particular value theory.

Value theories are accounting systems. Different value theories ask different questions and "enter" or begin their analysis with different "givens". A theory must always make a choice as to its entry point, ie, the concepts which it takes as given, and therefore, beyond question throughout the course of the analysis. The selection of a particular value theory by a theorist, is the theorist's, "coming-out", as it were, the declaration of his or her loss of theoretical innocence.

For instance, neoclassical value theory assumes that individuals are given an initial endowment of wealth (both human and non-human capital) and a fixed set of preferences, and that firms are given fixed technologies. On the basis of these initial assumptions, neoclassical economic theory attempts, among other things, to explain the exchange ratios by which commodititities trade.

Neo-Ricardian or Sraffian value theories explain how, given a fixed set of material input-output coefficients and along with the assumption that the competitive economy equalizes profit rates across firms, the material wealth of a society is divided into shares between the classes comprising that society.

Marxian value theory, on the other hand, differs from both neoclassical and Sraffian approaches to value theory. Marxian value theory begins by assuming that in all societies surplus labor is produced, appropriated and then distributed. Its focus, as developed by Resnick and Wolff (1982), is the everchanging set of fundamental and subsumed class processes in a social formation. From its entry point of class process, Marxian value theory proceeds to analyse the class relations of a society based on a definition of class in terms of socially necessary abstract labor-time, as opposed to a class knowledge of a society based on the distribution of wealth à la the Sraffian approach.

Value theories must necessarily abstract away from much of the complexity which characterizes the social totality. Value theories can not be all things to all theorists. It is in this sense that the choice of a value theory is a choice of a particular system of accounts. Milton Friedman understands this point well as he writes in his influential essay "Methodology of Positive Economics":

> "A completely 'realistic' theory of the wheat market would have to include not only the conditions directly underlying the supply and demand for wheat but also the kind of coins or credit instruments used to make exchanges; the personal characteristics of wheat-traders such as the color of each trader's hair and eyes, his antecedents and education, the number of members of his family, their characteristics, antecedents, and education, etc; the kind of soil on which the wheat was grown, its physical and chemical characteristics, the weather prevailing during the growing season; the personal characteristics of the farmers growing wheat and of the consumers who will use it; and so indefinitely" (1953, p. 32)

Marx understands equally well this point about value theories being accounting systems. In Chapter One of Volume I of Capital, he begins by stating "The wealth of societies in which the capitalist mode of production prevails appears as an 'immense collection of commodities' " (1976, p. 125). He then goes on to write that a commodity is a thing which satisfies human needs. He stresses that "Every useful thing is a whole composed of many properties ... The discovery of these ways and hence the manifold uses of things is the work of history ... The diversity of the measures for commodities arises in part from the diverse nature of the objects to be measured, and in part from convention" (Ibid., pp. 125 - 126).

These statements by Marx can be understood as his declaration of the need for a value theory. Here he is using a commodity as a metaphor for the social totality. Each commodity is a site which exists as the intersection of a variety of conditions of existence and can therefore, as with the elephant, be understood in a variety of different ways. Marx, of course, goes on in the three volumes of Capital to outline his value theory, i.e., his understanding of the commodity, by constructing a class knowledge of the commodity, and hence the social formation, based on the production, appropriation and distribution of surplus labor.

It is necessary to describe briefly the two approaches to value theory which will be contrasted below. The characteristics of these value theories will not be faithful to any particular theorist and may therefore unintentionally, and unfortunately, caricature some. The important point, however, is to bring out the differences between the two theories, not to render completely correctly all the nuances of any one value theory.

The surplus product approach to value theory, as will be used in this paper derives from the tradition beginning with Ricardo, and continuing through von Bortkiewicz, Leontief, Sraffa and the modern neo-Ricardians. It is also the value theory through which the Okishio Theorem is proved.

This approach takes as its object of analysis the production and distribution of the total material social product. For Ricardo,"To determine the laws which regulate this distribution, is the principle problem of Political Economy" (1977, p. 3). The surplus product approach begins by demonstrating that given (a) the technical input-output conditions of production and (b) the historically determined real wage, a surplus product can be determined. Here surplus product is defined as the total product minus the cost of material inputs and minus the total wage bill.

$$(1) \quad Z = \text{Total Product} - \text{Material Inputs} - \text{Wage Bill}$$

Z is the surplus product.

Of course, to determine the amount of surplus product as a scalar magnitude, total product, material inputs and the labor input must be denominated by the same terms. For the purpose of this paper the question of which particular denomination is the most appropriate one, is not a decisive issue. Therefore, let all commodities, both inputs and outputs, be denominated in terms of embodied socially necessary abstract labor-time.

The product rate of profit, π, can be expressed now as follows,

$$(2) \quad \pi = Z/(K + W)$$

where K is the amount of material inputs and W the amount of labor input, both denominated in units of abstract labor-time.

It is important to stress that all labor for the surplus product approach has the same conceptual status. All laboring activity, whether done by direct laborers, bookeepers, supervisors, or marketing personel, is creative of, or contributes to, the production of the surplus product. It is this point that is particularly important for differentiating between the surplus product and surplus labor appoaches to value theory.

The surplus labor approach to value theory begins by assuming that all production of material products is simultaneously the production of surplus labor. The surplus labor approach deploys a variety of concepts in order to analyse the different forms of the production, appropriation and distribution of surplus labor, i.e., the different forms of the fundamental and subsumed class processes. In contrast to the surplus product approach, this approach attributes different conceptual status to different types of labor. Productive labor is that labor which creates value and hence surplus value. All other labor is unproductive labor, that is, unproductive of surplus value. Even though unproductive labor does not create surplus value, it is a condition of existence of, i.e., constitutes in part, the production of surplus value. It is this important point which will be used extensively below in order to analyse the effect of the same initial technical change from the surplus product and surplus labor approaches to value theory respectively.

In order to bring out these differences, the following concepts of the surplus labor approach will be defined very generally here. They will be given a more specific definition below. Surplus labor, or surplus value, S, is the difference between total living productive labor and the labor needed to reproduce the productive laborers, or variable capital in value terms, V.

The value rate of profit can be defined as,

$$(3) \quad r = S/(C + V)$$

where C is constant capital, the socially necessary abstract labor time embodied in the material inputs.

Next, define the rate of exploitation as;

$$(4) \quad e = S/V$$

It is through the use of these concepts, appropriately modified below, that the contradictory effect of technical change can be 'seen' when analysed from the surplus labor approach to value theory.

<u>The Contradictory Effect of Technical Change on the 'Profit Rate'</u>

In order to isolate clearly the different effects that a given technical change will have on the product rate and the value rate of profit, it will be assumed in this paper that a technical change will only be introduced if it meets the following <u>viability condition</u>. A capitalist will adopt a new technique if and only if the new technique lowers unit total cost at the prevailing prices of production. Unit total cost is defined as the sum of unit material cost and unit total labor cost, where unit total labor cost equals the unit cost for both productive and unproductive labor. This is the viability condition used by the Okishio Theorem.

Now consider a technical change of the following type. Assume that there is a reorganization of the production process such that supervisory labor (one type of unproductive labor) is substituted for productive labor on a one for one basis. Assume also that as a result of this reorganization more output is produced. For simplicity, assume further that wages are equal, and do not change, for both supervisory and productive labor, and neither the amount nor mix of material inputs changes. Thus, as a result of this reorganization of production, unit total costs will fall and therefore in accordance with the viability condition this new technique of production will be adopted by an innovating capitalist. This type of technical change can be called a productive labor-saving, supervisory labor-using, capital-neutral technical change (PL-S, SL-U, C-N).

What will the effect of a PL-S, SL-U, C-N technical change be on the product rate of profit and on the value rate of profit? In order to answer this question two cases will be considered. First, it will be assumed that all commodities exchange at their prices of production, and that these prices remain constant. This assumption is equivalent to assuming a one commodity economy analogous to Ricardo's "corn-model" where all inputs and outputs are the same type of commodity. Second, commodities will still be assumed to exchange at their prices of production, but the prices will be allowed to adjust to the changed technique of production and the competitive pressure which equalizes the rate of profit (the product rate of profit in one case, and the value rate of profit in the other) across sectors. This assumption of equal profit rates is commonly used in linear price of production models, and is employed in the Okishio Theorem.

Case I Constant Prices of Production

Surplus product was defined above as total product minus total inputs. As total material inputs have remained constant and similarly total labor input has remained the same (albeit differently constituted), while output has risen, surplus product will therefore unambiguously rise. The product rate of profit was defined above in equation (2) as, $\pi = Z/(K + W)$. As surplus product, Z, unambiguously rises, while material costs, K, and total labor costs, W, remain the same, the product rate of profit also unambiguously rises.

What will be the effect of a PL-S, SL-U, C-N technical change on the production of surplus value and the value rate of profit? To calculate the effect on the magnitudes of these variables, it is necessary to recognize the contradictory effect such a technical change will have when accounted for from the surplus labor approach to value theory. Consider first the effect of this technical change on the production of surplus value. In general, surplus value can be written as, S = eV, where e is the rate of exploitation and V is variable capital. As $V = pbL^p$ (where L^p is productive labor, b is the bundle of wage goods, and p is the per unit price of production), the magnitude of V depends in part on the number of hours of productive labor employed , and in part on the money wage, pb, which is assumed to be constant. Thus as supervisory labor is substituted for productive labor there is a tendency for surplus value to fall. Obviously, the overall effect on surplus value will depend then on what happens to the rate of exploitation.

It is clear in this case that as supervisory labor replaces productive labor the rate of exploitation increases. This is so because fewer productive laborers are producing a greater total product. Assuming a given real wage, the productive laborers' necessary labor-time is reduced while at the same time surplus labor-time has risen. In effect, there has been an intensification of productive labor, resulting in the production of relative surplus value, thereby increasing the rate of exploitation. By assumption, as supervisory labor is increased, productive labor decreases and therefore, in this case, as productive labor decreases the rate of exploitation will rise. Thus, the overall effect on the production of surplus value depends on the relative sizes of the contradictory movements of the decrease in productive labor and the increase in the rate of exploitation.

Marx was aware of the importance of this intensification effect on the production of surplus value. In chapter 14 of of Volume III of Capital, the chapter on the counteracting tendencies to the tendency for the rate of profit to fall, he wrote that the forces which tend to lower the rate of profit, will tend to raise the rate of exploitation through the increased intensification of productive labor.

> "The mass of surplus-value that a capital of given size produces is the product of two factors, the rate of surplus value and the number of workers employed at this rate. With a given rate of surplus-value, therefore, it depends on the number of workers, and with a given number of workers it depends on the rate - in general, therefore, it depends on the product of the absolute size of the variable capital and the rate of surplus value. Now we have seen that the same factors that increase the rate of relative surplus value lower the amount of labor-power applied on average. It is evident, however, that this effect can be greater or less, depending on the specific proportions in which the antithetical movement takes place" (1981, p. 341).

Despite Marx's explicit consideration of this countertendency, the intensification effect has been largely ignored, or misunderstood, in the literature.

It is possible to derive the exact condition for a rise or fall in surplus value as productive labor decreases. To see this consider the following. Write surplus value as;

$$(5) \quad S = eV$$

It was argued above that the rate of exploitation is a positive function of supervisory labor. This can be written as $e = h(L^s)$, where $h' > 0$. It is assumed that $L^p + L^s = \bar{L}$, where $\bar{L}$ is the total amount of labor employed, which is assumed to be fixed. The rate of exploitation is therefore a negative function of productive labor and can be written as.

$$(6) \quad e = e(L^p) \quad \text{and } e' < 0$$

Variable capital is the product of the amount of productive labor hired and the money wage and can be written as;

$$(7) \quad V = wL^p$$

where w = pb, is the money wage and is assumed to be constant. By subsituting equations (7) and (8) into equation (5), surplus value can be expressed as.

$$(8) \quad S = e(L^p)wL^p$$

In order to see the overall effect of a reduction in productive labor on the production of surplus value, equation (8) can be differentiated with respect to productive labor, L^p. For simplicity it will be assumed here that a single capitalist enterprise only is under consideration, and therefore equation (8) can be totally differentiated with respect to L^p.

Differentiating equation (8) with respect to productive labor yields.

$$dS/dL^p = w(e'L^p + e)$$

$$= we(e'L^p/e + 1)$$

The ratio, $-e'L^p/e$, is the elasticity of the rate of exploitation with respect to changes in productive labor. As e' is negative by assumption,

$$(9) \quad dS/dL^p \gtrless 0 \quad \text{as} \quad E_{e,L^p} \lesseqgtr 1$$

where $E_{e,L^p} = - e'L^p/e$.

In words this condition says the following. As productive labor is reduced, surplus value falls (stays the same, rises) as the elasticity of exploitation with respect to changes in productive labor is less than (is equal to, is greater than) one. That is, surplus value falls (stays the same, rises) if the percentage decrease in productive labor is greater than (equal to, less than) the percentage increase in the rate of exploitation.

The value rate of profit can be expresed as;

$$(10) \quad r = e(L^p)wL^p / (C + wL^p)$$

by substituting equations (6) and (7) into equation (3). As productive labor appears in both the numerator and the denominator of the value rate of profit, it is necessary to differentiate equation (10) with respect to L^p in order to see the overall effect on the value rate of profit.

$$dr/dL^p = \frac{(e'wL^p + ew)(C + wL^p) - ew^2L^p}{(C + wL^p)^2}$$

$$= \frac{e'wL^pC + e'wL^pwL^p + ewC + ew^2L^p - ew^2L^p}{(C + wL^p)^2}$$

$$= \frac{we((e'L^p/e)(C + wL^p) + C)}{(C + wL^p)^2}$$

From this it can be seen that:

$$(11) \quad dr/dL^p \gtreqless 0 \quad \text{as} \quad E_{e,L^p} \lesseqgtr C/(C + V)$$

According to the condition expressed in equation (11), as productive labor falls (stays the same, rises), the value rate of profit falls (stays the same, rises) as the elasticity of exploitation with respect to productive labor is less than (equal to, greater than) the organic composition of capital expressed as C/(C + V). Thus, in contrast to the unambiguous result of a PL-S, SL-U, C-N technical change on the product rate of profit, the effect of the same technical change on the value rate of profit is conditional on the opposed movements of the increase in the rate of exploitation and the decrease in the absolute amount of productive labor employed. This contradictory movement is captured by Marx's concepts of relative and absolute surplus value. As pointed out above, the rise in the rate of exploitation leads to the creation of relative surplus value. The reduction of productive labor, however, leads to a reduction in the absolute number of hours worked which are creative of both value and surplus value. This is a diminuition in the production of absolute surplus value produced. The overall effect on the production of surplus value, and on the value rate of profit, as a result of a PL-S, SL-U, C-N technical change reflects these two ways to produce surplus value.

It should be noted that a necessary condition for the ambiguous movement of the value rate of profit is that the production of surplus value falls as productive labor falls. This can be seen by recognizing that for $dS/dL^p < 0$, i.e., for surplus value to increase as productive labor is reduced, it must be true that $E_{e,L^p} > 1$. And as C/(C + V) is always less than one, it will always be true that when $dS/dL^p < 0$, then $dr/dL^p < 0$. That is, if surplus value increases when productive labor falls the value rate of profit will necessarily rise. An examination of the definition of the value rate of profit reveals why this is so. The numerator, surplus value, rises while the denominator falls as variable capital is decreased and therefore, the ratio must necessarily rise.

The contradictory results of a PL-S, SL-U, C-N technical change when accounted for by the surplus labor approach to value theory can be summarized in the following table.

	$E < C/(C+V)$	$E > C/(C+V)$
$dS/dL^p > 0$	$dr/dL^p > 0$	$dr/dL^p < 0$
$dS/dL^p \leq 0$	$dr/dL^p < 0$	$dr/dL^p < 0$

Case II Flexible Prices of Production

In order to evaluate the effect of a PL-S, SL-U,C-N technical change on the product rate of profit and the value rate of profit respectively when prices of production are allowed to change and profit rates are equalized across sectors, a number of simplifying assumptions will be made. It will be assumed that (a) there are n-single product industries, (b) there is no fixed capital, (c) no scarce, non-produced means of production, i.e. no land, and (d) the wage bundle is held constant and is the same for all laborers. These are the assumptions of the Okishio Theorem.

The following notation will be used in the anlysis below.

$A = [a_{ij}]$ The matrix of physical commodity inputs per unit output where a_{ij} represents the amount of commodity i required to produce one unit of commodity j.

$L^p = [L_i^p]$ The row vector of productive labor inputs per unit output.

$L^s = [L_i^s]$ The row vector of supervisory labor (unproductive labor) inputs per unit output.

$b = [b_i]$ The column vector of commodities advanced per unit of productive or supervisory labor; represents the amount of commodity i consumed by a worker in return for an hour's labor-time.

$p = [p_i]$ The row vector of prices of production per unit output, where p_i is the quantity of socially necessary abstract labor time per unit output.

$r = S/C+V$ the general value rate of profit.

$\pi = Z/K+W$ the general product rate of profit.

$e = \frac{\sum e_i L_i^p}{\bar{L}}$ The social average rate of exploitation where e is a weighted average of the rates of exploitation, and $\bar{L}$ is the total productive labor hours worker in the economy, i.e., $\bar{L} = \sum L_i^p$

Consider first the effect of a PL-S, SL-U, C-N technical change from the perspective of the surplus product approach. There are n price of production equations which can be written as follows.

$$(13) \quad p = (1 + \pi)(pA + pb(L^p + L^s))$$

In all economically meaningful cases it can be assumed that A is a square, non-negative, irreducible matrix of input-output coefficients. Equation (13) can be rewritten as;

$$(14) \quad p = (1 + \pi)(pN)$$

where $N = A + b(L^p + L^s)$, the unproductive labor, augmented input matrix, with elements, $a_{ij} + b(L_i^p + L_i^s)$. From equation (14) it can be seen that p_i is a left eigen vector and the product rate of profit corresponds to the unique, maximum eigen value, α, where:

$$(15) \quad \pi = \frac{1 - \alpha}{\alpha}$$

Now, assume that a viable PL-S, SL-U, C-N technical change occurs in the sector producing commodity i. Let n_i^n represent the new column in the subsumed class augmented input matrix. According to the viability condition it must be true that:

$$p_i - p_i n_i^n (1 + \pi) > 0$$

It can be shown by an application of a Perron-Frobenius theorem that the maximum eigen value will fall as a result of this technical change and therefore the product rate of profit, as expressed in (15) will rise (see Bowles, 1981).

This is of course the same result as the Okishio Theorem, only now modified to include the explicit recognition of supervisory labor. That the Okishio Theorem would still hold, even with the explicit consideration of supervisory labor taken into account, should come as no surprise. All that has occurred is that the composition of the total labor input has been rearranged in such a way that unit total costs have fallen, resulting therefore in an increase in total surplus product. Once the competitive forces in the economy work themselves out, dividing this increased surplus product into equally weighted shares (the uniform rate of profit) across all sectors of the economy, the uniform rate of profit will unambiguously rise.

This is the economic logic involved in the use of the Perron-Fobenius Theorem. Bruce Roberts explains this logic as follows.

> "The use of Perron-Frobenius theorems accomplishes this task by, in effect, simultaneously dividing the physical output of each industry into two portions (in the same proportion across industries) which is expressible as a simple ratio of homogeneous physical goods. Equality of profit rates is achieved when "the counting system" (the structure of price relationships) is such that the capitals in each industry, in effect, 'get to keep' the same percentage of the physical output they produce. In other words, the homogeneous output of each industry is divided, in the same proportion, into on the one hand, a physical portion which is the precise exchange equivalent for the heterogeneous physical goods (means of production and means of subsistence) used up in production, and on the other hand, a physical portion which, as a residual, represents the surplus product of the industry, since it is the physical equivalent of the profit which can be realized when the output as a whole is sold" (1981, pp. 233-234).

Therefore, when the surplus product rises, due to the effect of the PL-S, SL-U, C-N, technical change, and once the Perron-Frobenius theorem is 'put to work' on the increased surplus product the general or uniform rate of profit must also rise.

In contrast, the recognition of productive labor and supervisory labor as conceptually distinct types of labor is critical in analysing the effect of the above technical change from the surplus labor approach to value theory. From this approach, only productive labor is creative of surplus value, while supervisory labor, instead, is merely constitutive of the process of surplus value production.

In order to evaluate the the change in the value rate of profit as a result of a PL-S, SL-U, C-N technical change I will modify the system of equations used by Wolff, Roberts, and Callari which they developed in an article on Marx's transformation of exchange values into prices of production (1982). There they summarize their understanding of Marx's value theory in and through the following set of equations.

$$(16) \quad p = (1 + r)(pA + pbL^P)$$

$$(17) \quad r = \frac{L^P X - pbL^P X}{pAX + pbL^P X}$$

$$(18) \quad V = L^P + pA$$

V is the unit value of producing each commodity, and X is the activity level. The other variables have the same meaning attributed to them above. For Wolff, Roberts and Callari, the major conceptual issue which these equations address is that both value and form of value are always simultaneously determined. This point is not of immediate concern here. Rather, the interest of this system of equations for the present paper is that they can be modified to explicitly include the presence of supervisory labor.

In their system, Wolff, Roberts and Callari implicitly assume that all labor is productive labor. It is this assumption which will be changed here by explicitly incorporating supervisory labor (unproductive labor) into their price of production framework. In effect, then, by so doing, a further transformation of the form of value will be carried out.

In order to accomplish this transformation rewrite the above systems of equations as follows.

$$(19) \qquad p^\circ = (1 + r)(p^\circ + p^\circ b(L^p + L^s))$$

$$(20) \qquad r = \frac{L^pX - p^\circ bL^pX}{p^\circ AX + p^\circ bL^pX}$$

$$(21) \qquad V^\circ = L^p + p^\circ A$$

This system of equations is modified by the explicit presence of supervisory labor in the price of production equations, and therefore p° and V° are the transformed or modified prices of production and values respectively.

In order to see the effect that a PL-S, SL-U, C-N technical change will have on the value rate of profit, it is sufficient to consider the system described by (19) and (20) only. Such a restriction results in a system where there are n prices of production and the value rate of profit to be determined by the n price equations and one profit rate equation.

Rewrite equation (19) and equation (20) as follows by substituting (8) into equation (20), as $L^pX - p^\circ bL^pX$ is equal to surplus value, S. For convenience, drop the superscript on the prices of production.

$$(21) \qquad p = (1 + r)(pA + pb(L^p + L^s))$$

$$(22) \qquad p_i = (1 + r)(pa_i + pb(L_i^p + L_i^s))$$

$$(23) \qquad r = \frac{e(L^p)pbL^p}{pA + pbL^p}$$

Equations (21) are now n-1 price equations. Equation

(22) is the price equation for the innovating sector and equation (23) is the value rate of profit.

Assume that a PL-S, SL-U, C-N technical change in the sector producing the ith commodity. As a result of this change the n prices of production and the value rate of profit will adjust to the changed production conditions. Assume that this is the only technical change to occur in the economy.

To calculate the overall effect of such a technical change on the value rate of profit, the system of equations, (21) - (22), will be partially differentiated with respect to L_i^p. Specifically, by taking the partial directional derivative with respect to L_i^p in the direction of the unit vector, the following differential equations result.

$$(24) \quad \frac{\partial p'}{\partial L_i^p} = (1+r)\left[\frac{\partial p'}{\partial L_i^p} A + \frac{\partial p'}{\partial L_i^p} b\left[L^p + L^s\right]\right] + N\left[pA + pb\left[L^p + L^s\right]\right]\frac{\partial r}{\partial L_i^p}$$

$$(25) \quad N\frac{\partial p_i}{\partial L_i^p} = (1+r)\left[\left[\frac{\partial p'}{\partial L_i^p} + \frac{\partial p_i}{\partial L_i}\right]a_i + \left[\frac{\partial p'}{\partial L_i^p} + \frac{\partial p_i}{\partial L_i^p}\right]b\left[L_i^p + L_i^s\right]\right] + \left[pa_i + pb\left[L_i^p + L_i^s\right]\right].$$

$$(26) \quad N\frac{\partial r}{\partial L_i^p} = \frac{\left[\left[e'p + e\left[\frac{\partial p'}{\partial L_i^p} + \frac{\partial p_i}{\partial L_i^p}\right]\right]bL^p + \gamma bep\right]\left[pA + pbL^p\right]}{\left[pA + pbL^p\right]^2} - \frac{\left[\left[\frac{\partial p'}{\partial L_i^p} + \frac{\partial p_i}{\partial L_i^p}\right]\left[A + bL^p\right] + \gamma pb\right]epbL^p}{\left[pA + pbL^p\right]^2}$$

where $$\frac{\partial p'}{\partial L^p_i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\partial P_j}{\partial L^p_i}$$

$$\gamma = \sum_{j=1}^{N} \frac{\partial L^p_j}{\partial L^p_i}$$

and as $$L^p = \left[L^p_1 \ldots . L^p_i \ldots . L^p_N \right]$$

therefore, $$\frac{\partial L^p}{\partial L^p_i} = \left[0, \ldots . 1, \ldots . 0 \right]$$

and, $$\sum_{j=1}^{N} \frac{\partial L^p_j}{\partial L^p_i} = \gamma = 1$$

By rearranging terms and simplifying the value rate of profit can be found by an application of Cramer's rule.

$$\begin{bmatrix} 1-(1+r)\left[A+b\left[L^p+L^s\right]\right] & 0 & N\left[pA+pb\left[L^p+L^s\right]\right] \\ -(1+r)\left[a_i+b\left[L^p_i+L^s_i\right]\right] & N\left[1-\left[(1+r)a_i+pb_i\left[L^p_i+L^s_i\right]\right]\right] & N\left[pa_i+pb\left[L^p_i+L^s_i\right]\right] \\ 0 & 0 & N\left[pA+pbL^p\right]^2 \end{bmatrix} \begin{bmatrix} \frac{\partial p'}{\partial L^p_i} \\ \frac{\partial p_i}{\partial L^p_i} \\ \frac{\partial r}{\partial L^p_i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Ne'pbL \\ \left[pA+pb \right. \\ + epbP \end{bmatrix}$$

The determinant of the coefficient matrix can be shown to be negative. The condition for an increase or decrease in in the value rate of profit due to a PL-S, SL-U, C-N technical change can be written.

$$(27) \quad \partial r / \partial L^p_i \gtrless 0 \quad \text{as} \quad E_{e,L^p_i} \lessgtr \frac{1}{n} \left(\frac{C}{C+V} \right)$$

This is a similar condition to the one derived previously in Case I under the assumption that prices of production remain constant as technology changes. The only difference here is that the organic composition of capital is multiplied by the factor, 1/n, where n is the number of sectors in the economy. When n equals one, as in Case I, condition (27) reduces to condition (11). However, as the number of sectors in the economy increases, the weighted organic composition of capital is reduced by the factor of 1/n. Therefore, for any given elasticity of exploitation with respect to productive labor, it will be more likely that this rate of exploitation will be greater than the weighted organic composition of capital as the economy grows. Thus, it will be more likely that the value rate of profit will rise as productive labor falls, i.e, it will be more likely that $\partial r/\partial L_i^p < 0$.

To see why this is so it must first be recognized that a necessary condition for the ambiguous result with respect to the value rate of profit is that the absolute amount of surplus value produced falls as productive labor falls. That is, $\partial S/\partial L_i^p > 0$. This condition was summarized above in Table 1. In other words, as $(C/(C+V))(1/n)$ is always less than one, for the contradictory result of equation (27) to hold it must be true that $E_{e,L_i^p} < 1$, which implies from Table 1 that $\partial S/\partial L_i^p > 0$.

The larger n is, the smaller the weighted organic composition of capital will be, ceteris paribus. Therefore, the larger n is, the more likely it will be that $E_{e,L_i^p} > (C/(C+V))(1/n)$, and the more likely it is that $\partial r/\partial L_i^p < 0$. That is, the more likely it is that the value rate of profit rises as productive labor falls. This reflects the fact that it is less likely that the increase in the rate of exploitation in the numerator of equation (23), the economy-wide weighted average of the rate of exploitation, will increase enough as a result of a change in the rate of exploitation in the innovative sector to outweigh the decrease in variable capital due to the fall in productive labor. However, as condition (27) implies, the overall effect on the value rate of profit is still ambiguous, even now under the assumption of flexible prices of production, resulting from a PL-S, SL-U, C-N technical change.

Conclusion

This paper has demonstrated that the rising rate of profit result of the Okishio Theorem does not depend exclusively on its explicit assumptions and the assumption of rational, capitalist behavior, but also on its use of a Sraffian value theory. The rate of profit which rises as a result of a viable technical change is the Sraffian rate of profit which is a measure of the surplus product produced in the economy. As was demonstrated above, when the rate of profit is calculated on the basis of a Marxian value theory, as a measure of surplus labor, the rate of profit can rise or fall in response to a viable technical change, even while maintaining all of the explicit assumptions of the Okishio Theorem.

This paper has sought to highlight the importance of value theory in economic analysis in general, and with respect to technical change in particular. This importance can be underscored by briefly considering the history of economic thought.

The history of economic thought can be divided into two traditions. On the one hand, there is the scarcity approach which characterizes neoclassical economics and which has dominated economic thought since the turn of the twentieth century. On the other hand, there is the surplus approach, represented by the Classicals and Marx during the nineteenth century, and more recently by the work of Sraffa and the renewed interest in Marx since the late 1960s.

This paper has compared two surplus approaches, that of Sraffa and Marx, to analysing the effect of technical change on their respective rates of profit. As was shown, contradictory results obtained. This should come as no surprise, as each value theory has a different understanding of surplus, and hence, rates of profit. The Sraffian approach's rate of profit is a measure of surplus product, where the surplus is calculated as a residual over all material inputs and all labor inputs. The Marxian rate of profit is a measure of surplus labor, where the surplus is calculated as a residual over material inputs and productive labor only.

The renewed interest in surplus approaches to economics, in contrast to the scarcity approach of neoclassical economics, has lead to much fruitful research in recent years. However, it should not be forgotten that the surplus approach is not a homogeneous tradition. In particular, there are two major areas of disagreement within this tradition which have important theoretical and political implications.

First, what is it that the surplus is a measure of? Is it surplus labor which is being measured? Or surplus product? Or surplus dollars? It may be the case that these measures are proportional to one another and therefore the respective profit rates will be equal, but that need not always be true. Second, how is the surplus to be calculated? Over all inputs? Should productive labor and unproductive labor be separated out? What about taxes? These are critical questions for theoretical and political anlysis. This paper has demonstrated that importance for the analysis of technical change.

It is incumbent for those working within the surplus tradition that the conflation of different definitions of surplus be avoided. Otherwise, there will be much fruitless spinning of wheels in the attempt to offer alternative economic*s* to the dominant neoclassical paradigm.

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